

Revisiting the Analysis of Early Access to Algebra under the Counterfactual Framework

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Background

- June 2008, California State Board of Education enacts algebra content testing policy for 8th graders
- Only about half of all CA 8th graders currently take algebra
- Research supports positive relationship between taking algebra and mathematics achievement (e.g., Gamoran & Hannigan, 2000; Ma, 2005; Smith, 1996)
- Some fear unprepared students will encounter difficulties (Loveless, 2008)

Counterfactual Framework Overview

- Causal effects defined by potential outcomes at the unit of analysis:
 - $\bar{\delta}_i = y_i(i) - y_c(i)$
- Average treatment effect (ATE):
 - $E[\bar{\delta}_{ATE}] = E[Y_t] - E[Y_c]$
- Average treatment effect for the treated (ATT):
 - $E[\bar{\delta}_{ATT}] = E[Y_t | X=1] - E[Y_c | X=1]$
- Average treatment effect for the control (ATC):
 - $E[\bar{\delta}_{ATC}] = E[Y_t | X=0] - E[Y_c | X=0]$
- **But $E[Y_c | X=1]$ and $E[Y_t | X=0]$ not observable**

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Counterfactual Framework Overview

- Under assumptions of selection independence and unit homogeneity:
 - $E[Y_t | X=1] = E[Y_t | X=0]$
 - $E[Y_c | X=1] = E[Y_c | X=0]$
- and ...
 - $E[\bar{\delta}_{ATE}] = E[Y_t | X=1] - E[Y_c | X=0]$
 - $E[\bar{\delta}_{ATE}] = E[\bar{\delta}_{ATT}] = E[\bar{\delta}_{ATC}]$

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Counterfactual Framework Overview

- But assumption of selection independence rarely holds outside of a randomized study
- To estimate effects:
 - Fall back on assumption of conditional independence
 - Can condition on a set of covariates (Z) using statistical methods (e.g., regression, matching, stratification, etc.)
- And assumption of unit homogeneity is rarely investigated

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Back to the Background

- Past research speaks to the more general question of whether the average student (ATE), or those who experience early access to algebra (ATT), benefit
- Selection process suggests “treated” and “non-treated” students are very different
- Current policy debate raises question about effect of algebra for the “non-treated” students (ATC)
- If effect heterogeneity exists, $ATT \neq ATC$

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Research Objective

- Test sensitivity of prior regression-based research on algebra by locating the analysis within the counterfactual framework
 - ▣ Use propensity score methods instead of traditional regression methods
 - ▣ Focus on heterogeneity of causal effects
 - ▣ Test for a dropout effect
(Not part of this presentation but in paper)

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Propensity Score Methods

- Compare OLS regression-based effect estimates to weighted OLS regression-based effect estimates following Morgan & Todd (2008)
 - ▣ ATT weight
 - for $X=1$, $w_i = 1$
 - for $X=0$, $w_i = p_i / (1-p_i)$
 - ▣ ATE weight
 - for $X=1$, $w_i = 1 / p_i$
 - for $X=0$, $w_i = 1 / (1-p_i)$
 - ▣ ATC weight
 - for $X=1$, $w_i = (1-p_i) / p_i$
 - for $X=0$, $w_i = 1$
 - ▣ Where p_i is the estimated conditional probability of treatment assignment ($X=1$), i.e., the propensity score

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Propensity Score Methods

- Also estimate ATT & ATC using two STATA propensity score matching algorithms
 - ▣ *psmatch2* (Leuven and Sianesi, 2003)
 - 1-to-4 nearest neighbor matching w/ replacement and caliper of 0.05
 - ▣ *nmmatch* (Abadie et al, 2004)
 - 1-to-4 nearest neighbor matching w/ replacement and common support

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Data Source

- National Education Longitudinal Study (NELS:88)
 - ▣ Panel data for 8th graders in 1988 and 12th graders in 1992
 - ▣ Examine selection process based on base year cohort (N=18,020) use panel data for 12th grade effect estimates (N=9,951)
 - ▣ NELS used a two-staged stratified probability design
 - Use STATA survey estimation procedure and sampling design weights to adjust standard errors for survey design
 - Can multiply propensity score-based weights and survey design weights

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Data Source

- Treatment Variable: Take Algebra in 8th grade
 - ▣ Did you attend an algebra (or other advanced math) class at least once a week during the current school year?
 - ▣ About 40% of students took algebra in 8th grade
- Dependent Variable: Mathematics achievement in 12th grade
 - ▣ Standardized scores from NELS 2nd follow-up mathematics test
 - ▣ Scores ranged from 30 to 70, mean = 52, sd = 10

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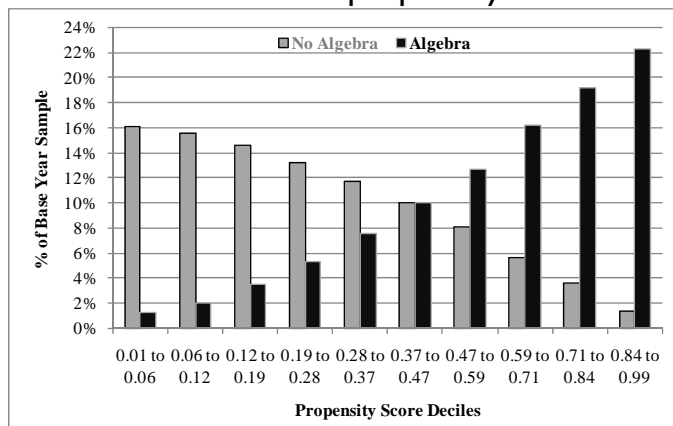
Data Source

- Covariates (Z)
 - ▣ 3 student characteristics
 - ▣ 6 family characteristics
 - ▣ 7 student academic history indicators
 - ▣ School-level % 8th graders in algebra

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Propensity Score Model

- $\log(p_i / (1-p_i)) = \beta_0 + Z_i\beta_1 + e_i$ (see Table 1)
- Distribution of estimated propensity scores



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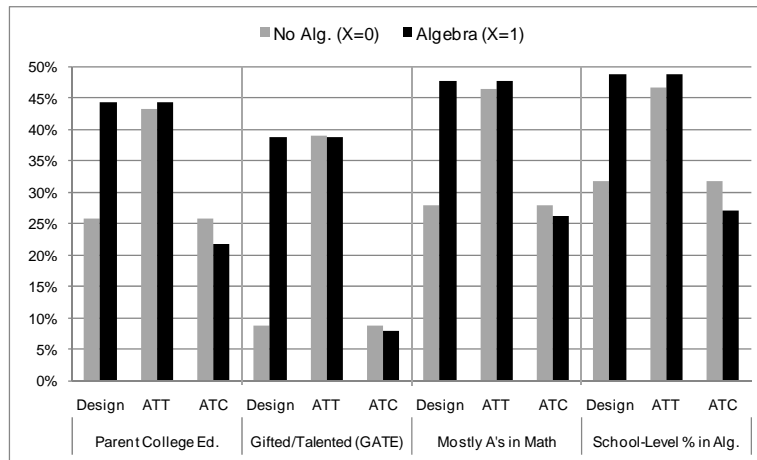
Propensity Score Model

- Bias Reduction through Propensity Score Weighting
 - Average Standardized Bias
 - Before P-Score Weighting: 0.22
 - ATT Weighting: 0.02
 - ATC Weighting: 0.11
 - See Table A.3

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Propensity Score Model

□ Bias Reduction through Propensity Score Weighting



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Effect Estimates

□ Weighted Regression Models ($Y_i = \beta_0 + X_i\beta_1 + Z_i\beta_2 + e_i$)

- Model 1: unconditional
- Model 2: conditions on 3 student characteristics
- Model 3: full conditional model

	Design Weight	ATE Weight	ATT Weight	ATC Weight
Model 1	8.073 ** (0.309)	2.102 * (0.986)	4.299 ** (0.493)	0.976 (1.265)
Model 2	6.248 ** (0.272)	2.628 ** (0.586)	4.051 ** (0.418)	1.887 * (0.730)
Model 3	3.961 ** (0.262)	3.346 ** (0.320)	3.913 ** (0.337)	3.112 ** (0.387)

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Effect Estimates

□ Matching Estimates (*nnmatch*)

□ By propensity score level

	ATE	ATT	ATC
Main Effect	3.350 ** (0.354)	2.908 ** (0.404)	3.667 ** (0.501)
Low Propensity Deciles	3.102 ** (0.748)	1.903 (1.081)	3.278 ** (0.807)
Medium Propensity Deciles	3.956 ** (0.422)	3.553 ** (0.514)	4.252 ** (0.506)
High Propensity Deciles	3.054 ** (0.533)	2.775 ** (0.591)	4.177 ** (0.721)

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Conclusions

- Evidence supports an “algebra for all” policy
 - Statistically significant positive effects on 12th grade mathematics performance
 - Effect estimates fairly consistent for both treated and untreated students (homogeneous effects)
- Relevance of research to current policy debate still limited
 - Assumption of conditional independence questionable
 - Stable-unit-treatment-value assumption (SUTVA) questionable

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Table 1. Logistic Regression Model Prediction of 8th Grade Algebra Selection

	Coef.	Std. Err.	p-value	Odds Ratio	95% Conf. Interval
Student Characteristics					
Female	-0.169	0.043	0.000	0.845	0.777 - 0.918
Ethnicity (White Omitted):					
Black	-0.088	0.066	0.185	0.916	0.804 - 1.043
Hispanic	-0.010	0.087	0.907	0.990	0.834 - 1.174
Asian/PI	0.268	0.112	0.017	1.307	1.049 - 1.628
Native American	0.105	0.182	0.564	1.111	0.777 - 1.588
LEP	-0.284	0.175	0.106	0.753	0.533 - 1.062
Family Characteristics					
Number of Siblings	-0.022	0.014	0.110	0.978	0.951 - 1.005
Highest Parent Education (HS Grad Omitted):					
Unknown	-0.135	0.093	0.147	0.874	0.729 - 1.048
Less Than H.S.	-0.193	0.096	0.044	0.824	0.683 - 0.995
Junior College	0.132	0.060	0.028	1.141	1.015 - 1.283
College	0.157	0.066	0.018	1.170	1.028 - 1.332
Parent School Involvement	0.213	0.069	0.002	1.237	1.081 - 1.416
Single-Parent Family	-0.049	0.058	0.399	0.952	0.850 - 1.067
Family SES	0.121	0.043	0.005	1.129	1.038 - 1.227
English-Only Family	-0.039	0.073	0.592	0.962	0.833 - 1.110
Academic History					
GATE	1.737	0.066	0.000	5.681	4.994 - 6.464
Ever Retained in a Grade	-0.512	0.067	0.000	0.599	0.525 - 0.683
Math Grades (Mostly C's Omitted):					
Mostly A's	0.827	0.069	0.000	2.286	1.996 - 2.618
Mostly B's	0.360	0.061	0.000	1.433	1.272 - 1.615
Mostly D's	-0.003	0.110	0.979	0.997	0.803 - 1.238
Mostly F's	-0.123	0.194	0.527	0.884	0.604 - 1.294
Ungraded	-0.230	0.629	0.715	0.795	0.231 - 2.728
Want to go to College	0.477	0.061	0.000	1.611	1.429 - 1.817
Misbehaving	-0.264	0.049	0.000	0.768	0.698 - 0.845
Poor School Work	-0.277	0.083	0.001	0.758	0.644 - 0.893
Attendance Issues	-0.062	0.081	0.447	0.940	0.802 - 1.102
School-Level % in Alg. (Log)	1.765	0.042	0.000	5.841	5.375 - 6.346
Constant	0.306	0.140	0.029		

Note: Results reflect use of survey design weights (df=958) for the NELS 1988 base year sample (Unweighted N=18,020).

Table A.3. Comparison of Covariate Means for No Algebra and Algebra Groups based on Propensity Score Weighting, Base Year Sample (N=18,020)

	Design Weighted			ATT Weighted		ATC Weighted	
	No Alg.	Alg.	Bias	No Alg.	Bias	Alg.	Bias
Female	0.530	0.517	0.026	0.530	0.025	0.485	0.090
Ethnicity: White	0.741	0.765	0.055	0.747	0.042	0.690	0.120
Ethnicity: Black	0.115	0.099	0.053	0.106	0.023	0.106	0.027
Ethnicity: Latino	0.106	0.077	0.102	0.091	0.048	0.165	0.202
Ethnicity: Asian/Pac. Is.	0.025	0.049	0.130	0.047	0.012	0.023	0.012
Ethnicity: Nat. Am.	0.012	0.010	0.022	0.009	0.008	0.017	0.041
Limited English Proficient (LEP)	0.019	0.011	0.066	0.014	0.022	0.038	0.162
Number of Siblings	2.351	2.112	0.154	2.168	0.036	2.302	0.032
Highest Parent Educ: Unknown	0.084	0.059	0.098	0.064	0.021	0.090	0.026
Highest Parent Educ: Less Than H.S.	0.116	0.048	0.253	0.051	0.011	0.148	0.118
Highest Parent Educ: High School	0.317	0.217	0.226	0.213	0.010	0.349	0.074
Highest Parent Educ: Junior College	0.226	0.233	0.016	0.238	0.013	0.195	0.074
Highest Parent Educ: College	0.258	0.443	0.396	0.434	0.020	0.219	0.084
Parent School Involvement	0.847	0.911	0.196	0.911	0.002	0.825	0.069
Single-Parent Family	0.222	0.179	0.108	0.183	0.010	0.259	0.091
Family SES Composite	-0.193	0.169	0.492	0.139	0.040	-0.345	0.207
English-Only Family	0.837	0.831	0.015	0.819	0.032	0.793	0.118
Gifted/Talented (GATE)	0.088	0.387	0.778	0.390	0.007	0.080	0.020
Ever Retained in a Grade	0.205	0.081	0.367	0.087	0.018	0.316	0.327
Math Grades: Mostly A's	0.279	0.476	0.417	0.466	0.023	0.263	0.032
Math Grades: Mostly B's	0.368	0.352	0.034	0.357	0.010	0.336	0.067
Math Grades: Mostly C's	0.250	0.134	0.300	0.137	0.009	0.257	0.018
Math Grades: Mostly D's	0.073	0.029	0.205	0.031	0.008	0.098	0.119
Math Grades: Mostly F's	0.028	0.008	0.158	0.008	0.003	0.036	0.066
Math Grades: Ungraded	0.003	0.001	0.036	0.001	0.002	0.009	0.166
Student Wants to go to College	0.753	0.909	0.433	0.907	0.005	0.711	0.118
Sent to Office for Misbehaving	0.379	0.266	0.243	0.267	0.003	0.449	0.152
Sent to Office for Poor School Work	0.110	0.055	0.204	0.060	0.018	0.151	0.149
Parents Notified about Attendance	0.125	0.073	0.177	0.093	0.071	0.160	0.121
School-Level % in Algebra	0.317	0.488	0.926	0.468	0.111	0.270	0.255
Overall Mean Standardized Bias			0.223		0.022		0.105
Propensity Score	0.265	0.592		0.581		0.213	

Note: Bias reflects absolute difference between means of the No Algebra (control) group and Algebra (treatment) group divided by the pooled standard deviation. ATT weight bias compares weighted control group means to design weighted treatment group means. ATC weight bias compares weighted treatment group means to design weight control group means. See text for discussion of weights.